EPFL

Plastic hinge method

Shortcomings of force-based design

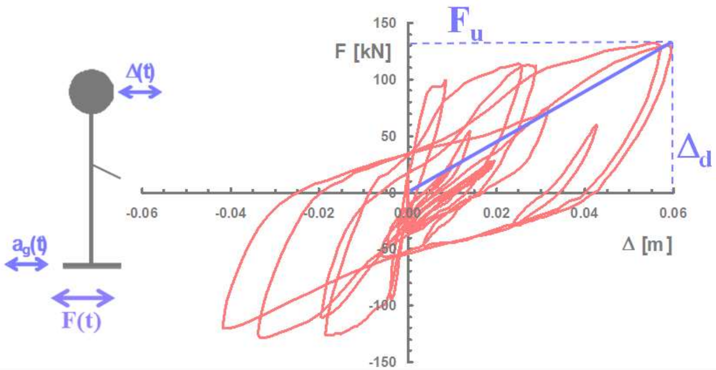
Seismic Engineering

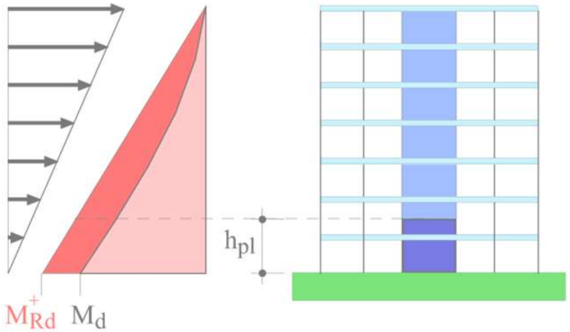
Dr. Francesco Vanin

Course objectives

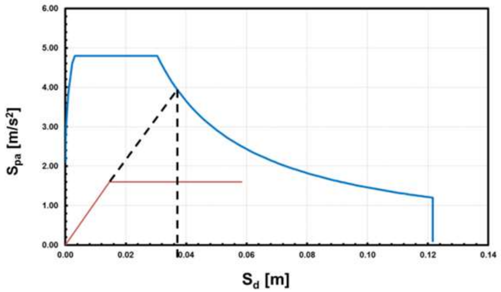
Know typical failure modes of structures during earthquakes.

Know how to estimate the peak forces and displacements of structures subjected to earthquakes.





Know how to design new buildings with reinforced concrete walls.



Know the basic elements of a displacement-based evaluation of existing structures.

Lecture objectives

- Learn to use the plastic hinge method for computing the Force-displacement capacity of RC sections
- Understand difference between local and global ductilities
- Understand important shortcomings of force-based design

Background:

- RC section analysis

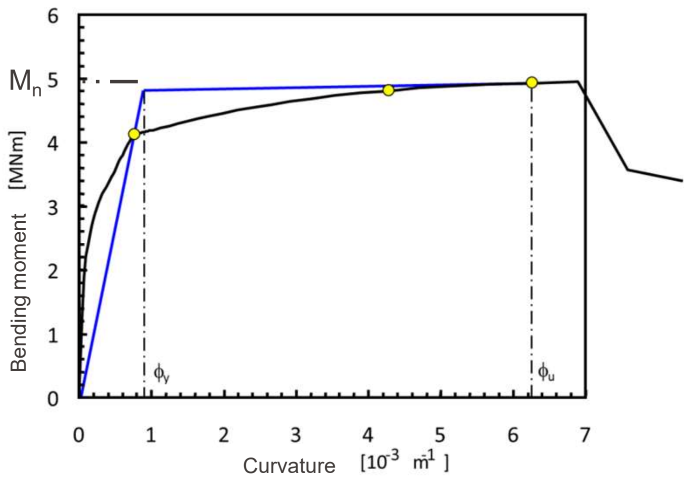
Content

- Computation of the inelastic displacement capacity by means of the plastic hinge method
  - Analysis of moment-curvature relationship
  - Plastic hinge method
  - Relationship between local and global ductilities

- Shortcomings of force-based design
  - How will force-based structures perform during an earthquake?
  - Problem 1: Force-based design needs as input an estimate of the initial period of the building
  - Problem 2: Using the same q-factor does not lead to the same performance
  - Problem 3: Force-based design is based on elastic analysis

Plastic hinge analysis

Response of an RC section

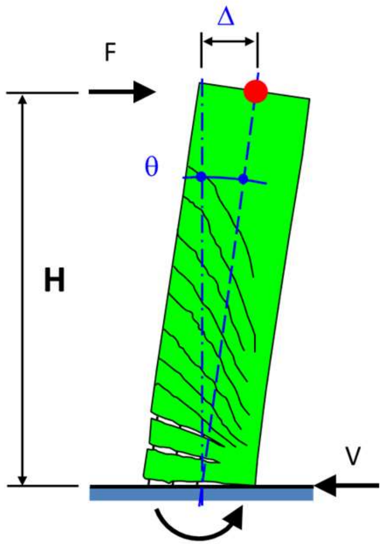


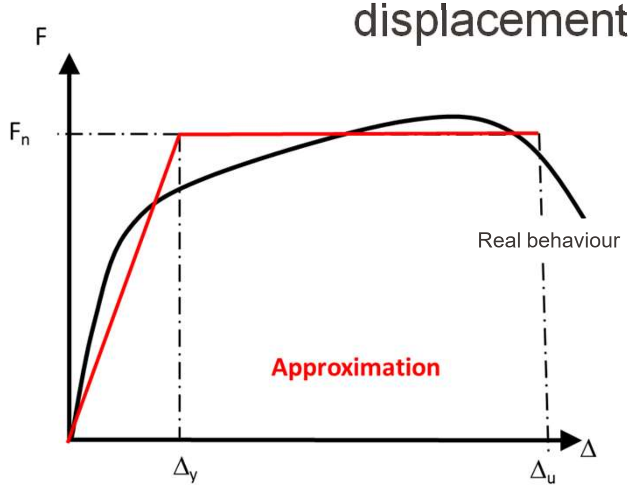
Bilinear approximation of moment-curvature relationship for section that plastifies: M_n, ϕ_y, ϕ_u

Plastic hinge analysis

Bilinear approximation of force-displacement relationship F_n, Δ_y, Δ_u

Response of an RC member



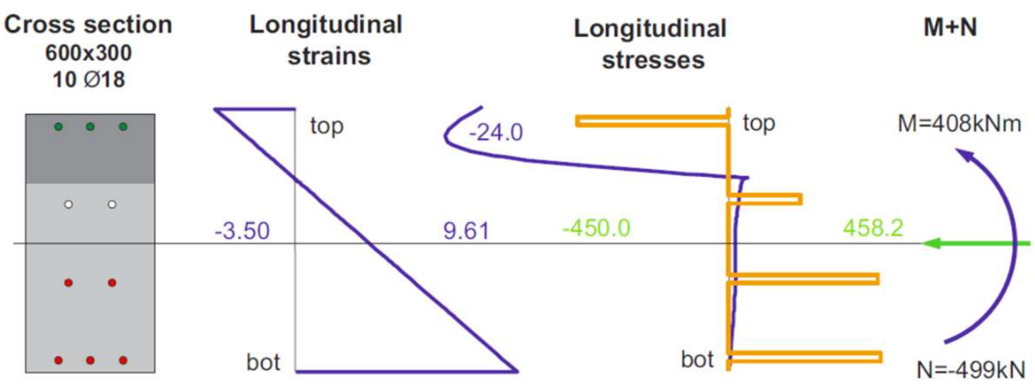


Objective: Estimate with simple means the inelastic displacement capacity

Moment-curvature analysis

Moment-curvature analysis

= Computation of moment for increasing curvature and constant axial force



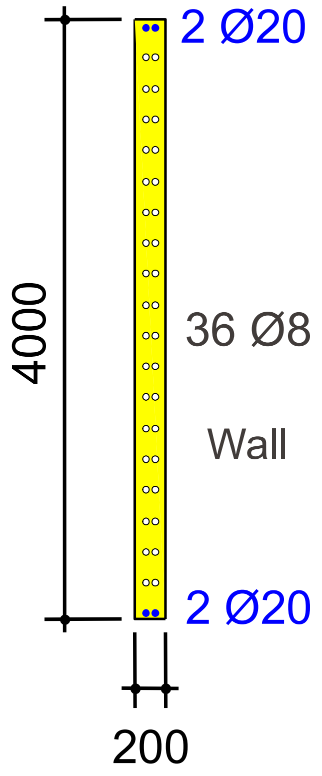
Required input:

- Geometry of RC section
- Stress-strain relationships of confined and unconfined concrete and of longitudinal reinforcement bars

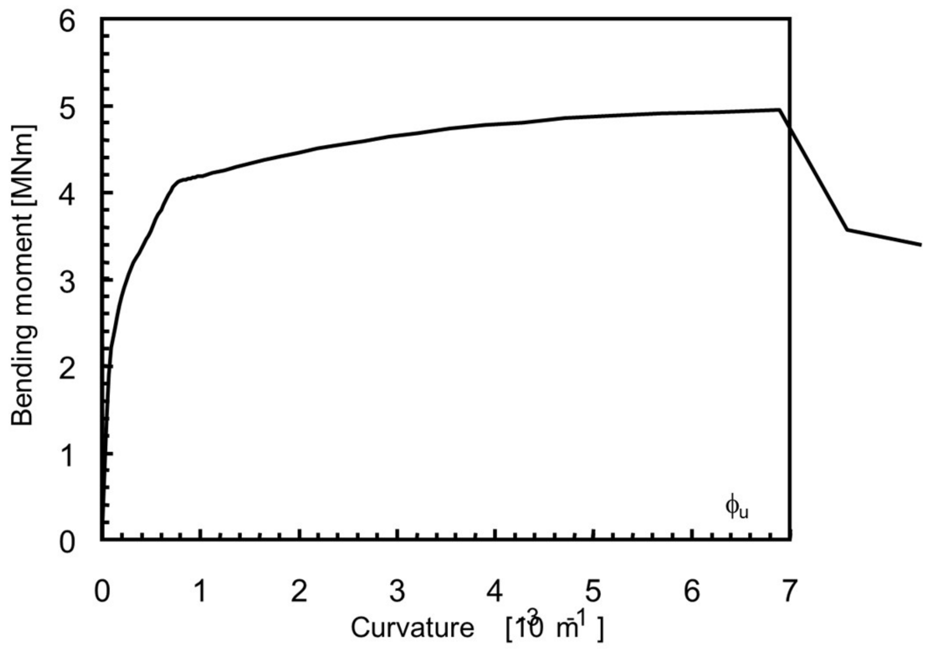
Assumptions:

- Linear strain profile («plane sections remaining plane», Bernoulli)
- Perfect bond between reinforcement and concrete → Strain only dependent on the distance to neutral axis

Moment-curvature relationship



N = -1099 kN



@ A. Dazio

Moment-curvature analysis

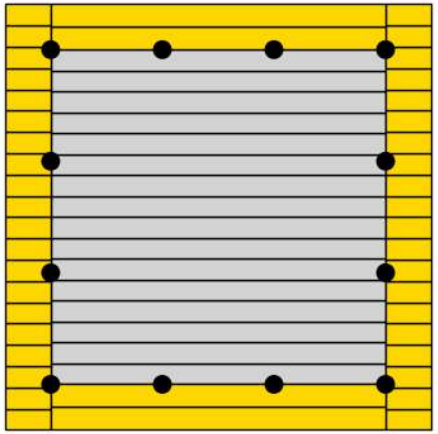
Procedure: Moment-curvature analysis for a constant axial force N

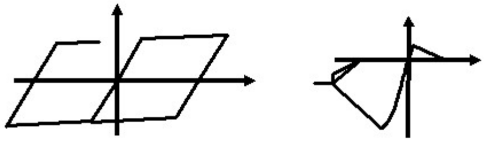
- Divide section into layers
- Determine for each layer the area of unconfined concrete, confined concrete and reinforcing steel
- Define stress-strain relationship for unconfined concrete, confined concrete and reinforcing steel

- Choose a strain of the extreme compression fibre ε_c
- Assume a neutral axis depth c
- Compute strain at the centreline of each layer
- Calculate for each layer the concrete and reinforcement stress (from stress-strain relationship)
- Calculate for each layer the concrete and reinforcement forces
- Check whether the sum of all concrete and reinforcement forces gives the axial force N
- If not, modify c and iterate until agreement is satisfactory.
- Compute the moment M and the curvature ϕ

- Increase the strain of the extreme compression fibre ε_c to compute further points of the moment-curvature relationship

Moment-curvature analysis

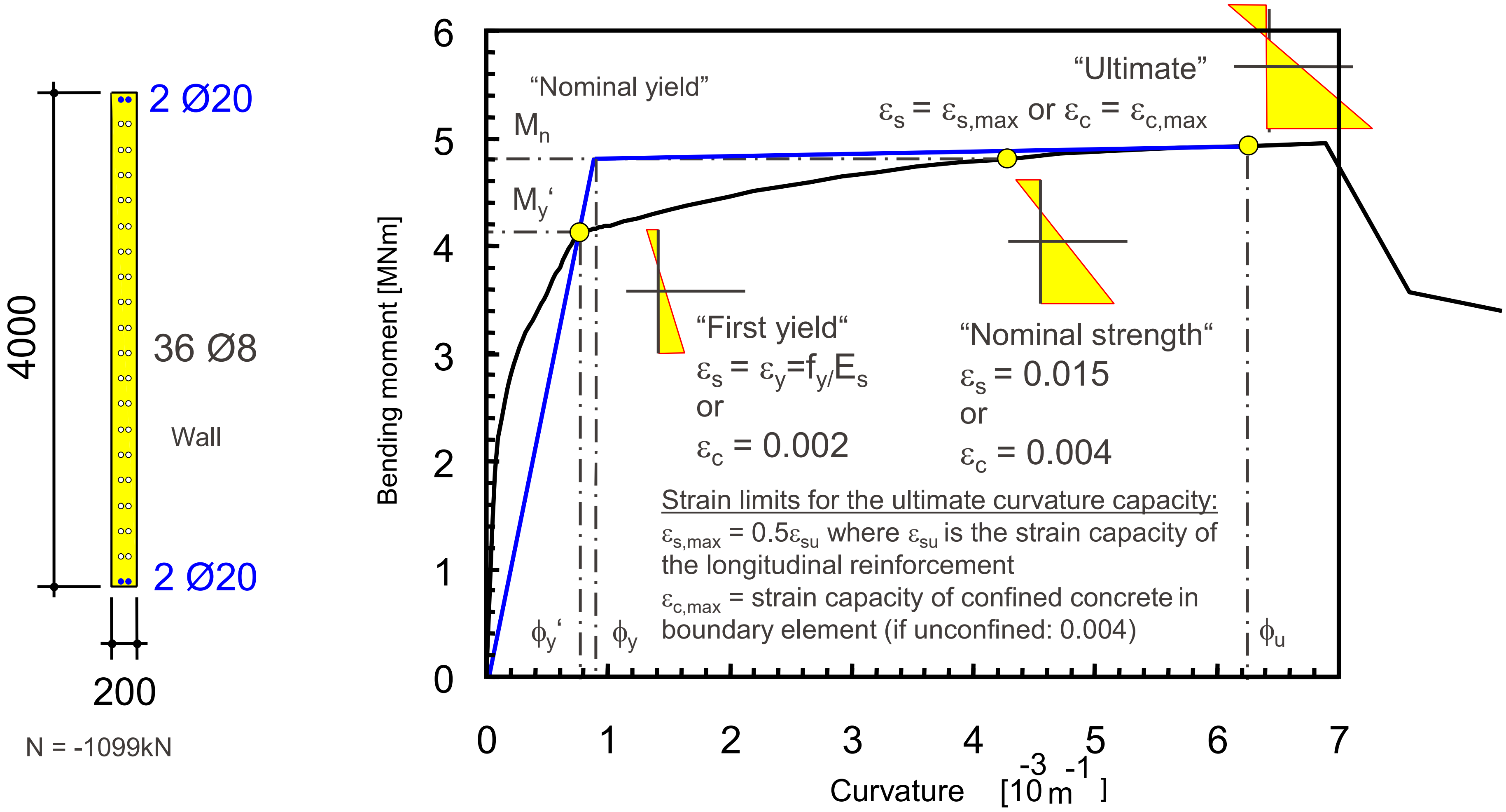




material stress-strain

Moment-curvature analysis

Moment-curvature analysis – bilinear approximation



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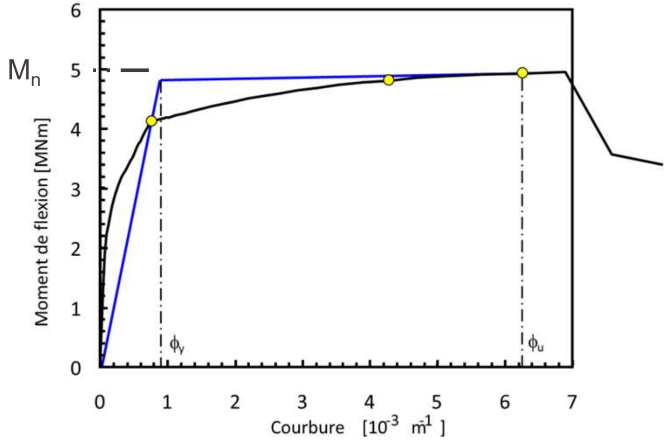
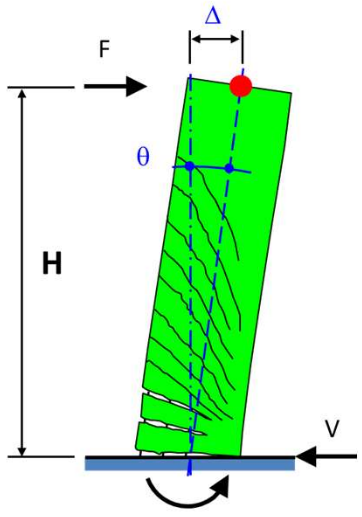
Plastic hinge analysis

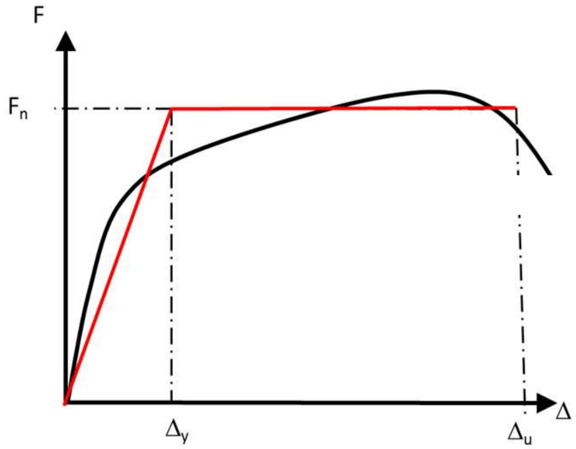


Bilinear approximation of moment-curvature relationship for section that plastifies:
M_n, ϕ_y, ϕ_u

Plastic hinge analysis

Bilinear approximation of force-displacement relationship F_n, Δ_y, Δ_u





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Plastic hinge analysis

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Plastic hinge method

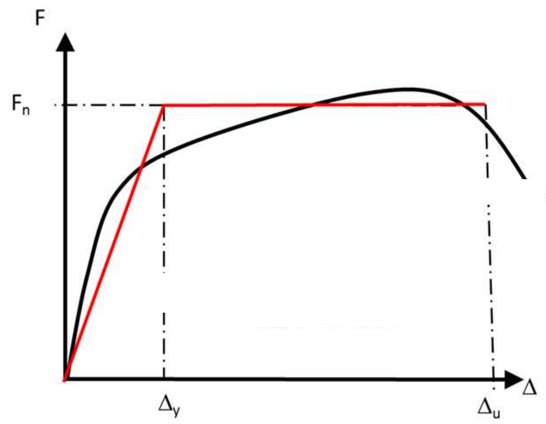
The plastic hinge method is a very simple model for estimating the inelastic deformation capacity of

- Slender, ductile RC walls
- RC columns and steel columns

→ i.e. elements that form a ductile flexural mechanism and where shear deformations do not play a significant role

The bilinear force-displacement response can be constructed by computing the force-displacement relationship from:

- the moment-curvature relationship of the section that plastifies,
- the geometry of the structural element, and
- the plastic hinge length.



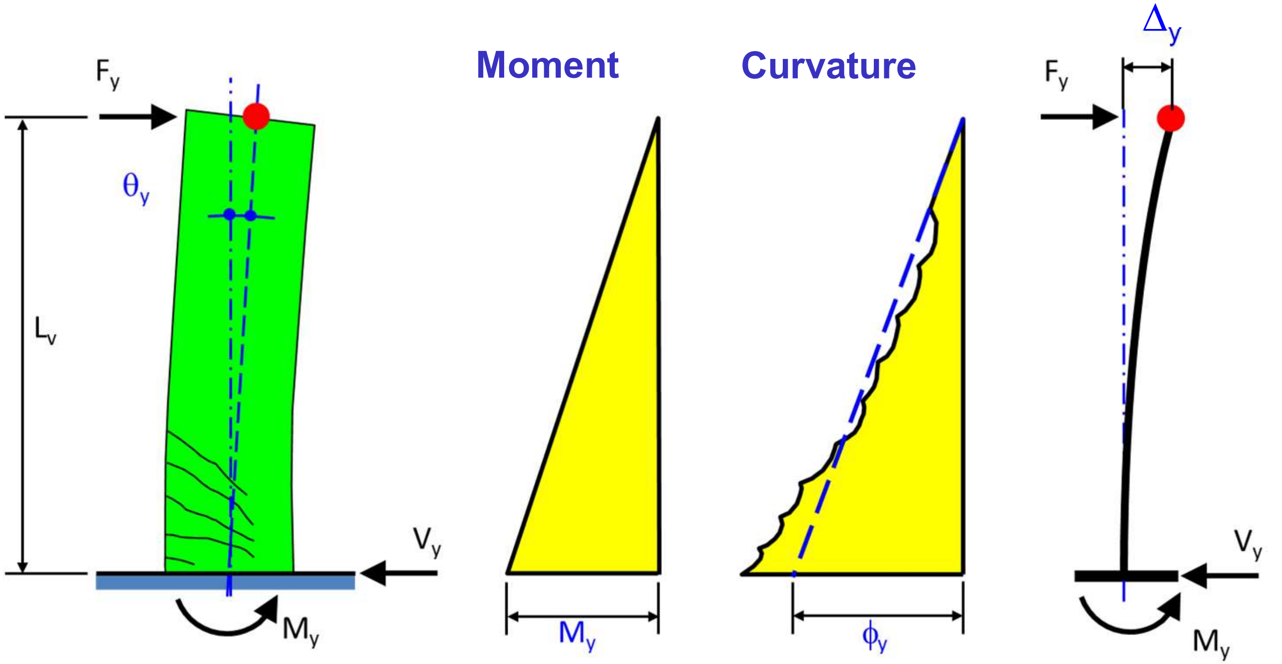
Three quantities that determine bilinear force-displ. relationship:

- Force capacity F_n
- Yield displacement Δ_y
- Ultimate displacement capacity Δ_u

Plastic hinge analysis

Force capacity F_y & Yield displacement Δ_y

- Assume a linear curvature profile over the shear span L_v



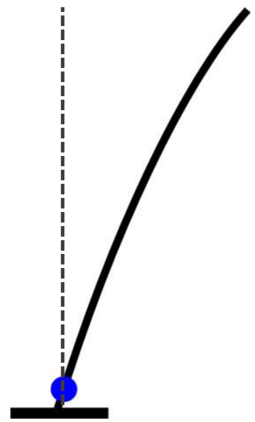
$$\Delta_y = \frac{\varphi_y}{3} \cdot L_v^{\ 2}$$

Plastic hinge analysis

Ultimate displacement Δ_u

Plastic hinge model

- Hypothesis: The inelastic deformation is concentrated in a plastic hinge. The rest of the structure remains elastic.
- The plastic rotation capacity of the plastic hinge is estimated from the ultimate curvature ϕ_u and the plastic hinge length L_p.



Assumptions:

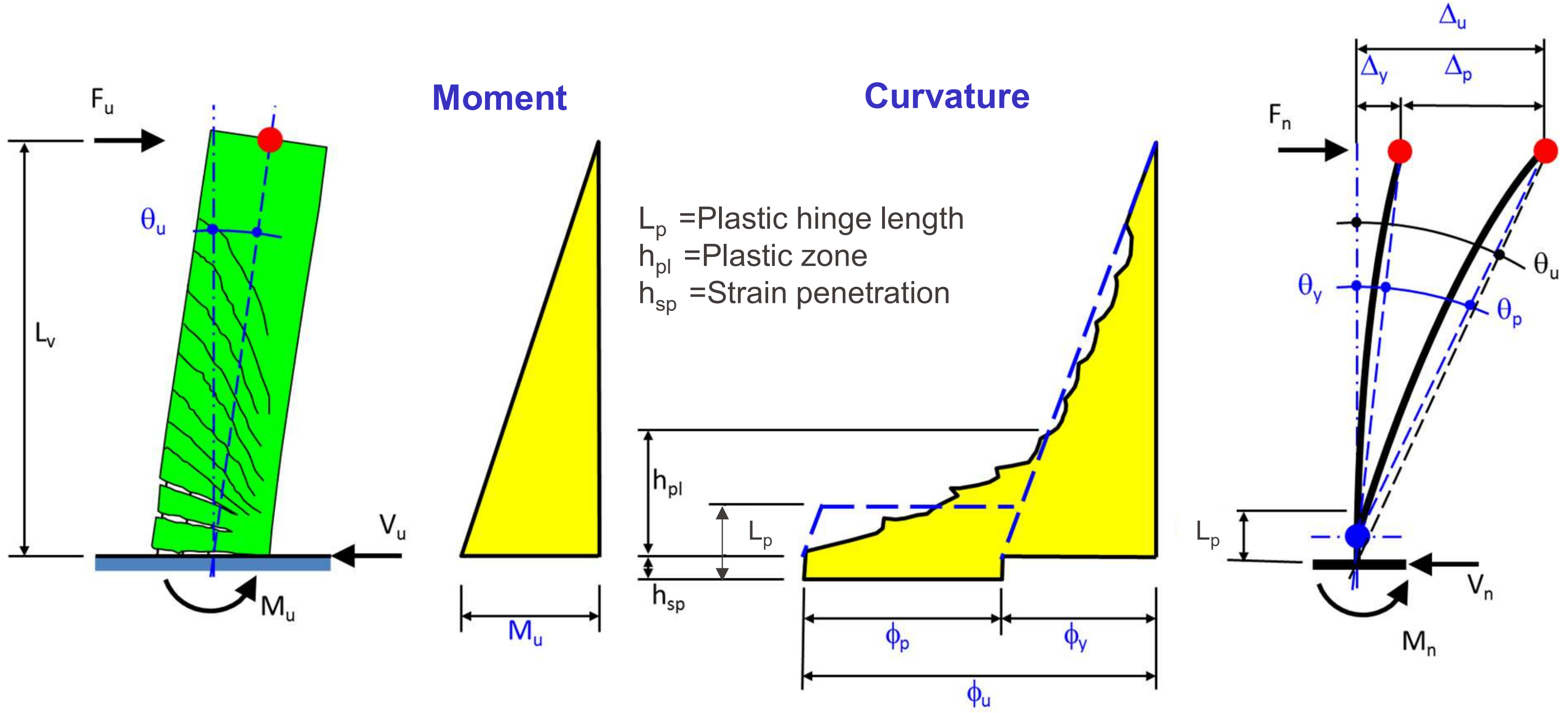
- The method accounts only for the flexural deformations and neglects the shear deformations.

→ The method gives good approximations of the actual displacement capacity if the shear deformations are relatively small.

- Several methods have been proposed to account for shear deformations but they are not uniformly applied.

Plastic hinge analysis

Ultimate displacement Δ_u



$$\Delta_u = \frac{\varphi_y}{3} L_v^{\,2} + \left(\varphi_u - \varphi_y\right) \cdot L_p \cdot (L_v - 0.5 \cdot L_p)$$

Plastic hinge analysis

Length of the plastic hinge L_p

Semi-empirical equations

PCK07:

$$L_p = kL_v + \alpha l_w + L_{sp} \geq 2L_{sp}$$

$$k = 0.2\left(\frac{f_t}{f_s} - 1\right) \leq 0.08 \qquad L_{sp} = 0.022 d_b f_s$$

SIA 269/8:

$$L_p = a_{st}\left(0.08L_v + L_{sp}\right) \geq 2a_{st}L_{sp}$$

$$L_{sp} = 0.022 d_b f_s$$

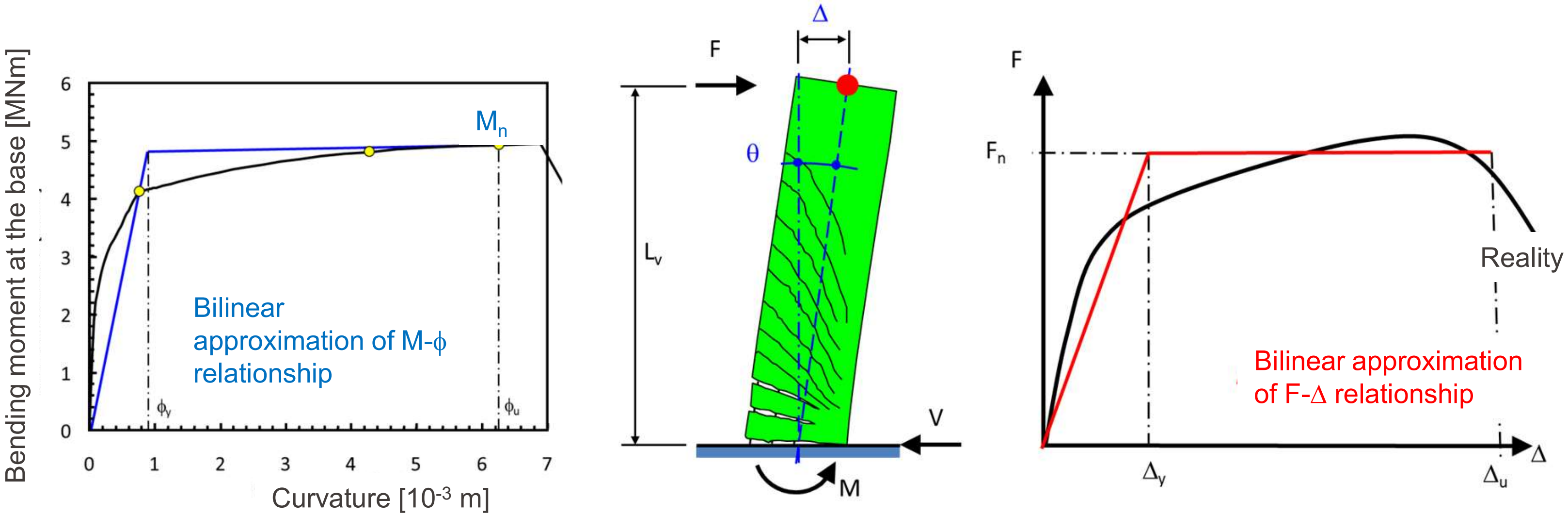
$f_t/f_s < 1.15$: $\quad a_{st} = 0.8$

$f_t/f_s \geq 1.15$: $\quad a_{st} = 1.0$

L_v	= M_{base}/V_{base} (Shear span)
l_w	Length of the wall
L_{sp}	Strain penetration length into foundation
k	Coefficient that accounts for the hardening of the longit. reinforcement bars
f_t	Tensile strength of reinforcement bars
f_s	Yield strength of reinforcement bars
d_b	Largest diameter of a longitudinal bar in the plastic zone
α	Coefficient that accounts for tension shift:
	Walls: α=0.1 Columns: α=0

Plastic hinge analysis

Force-displacement relationship



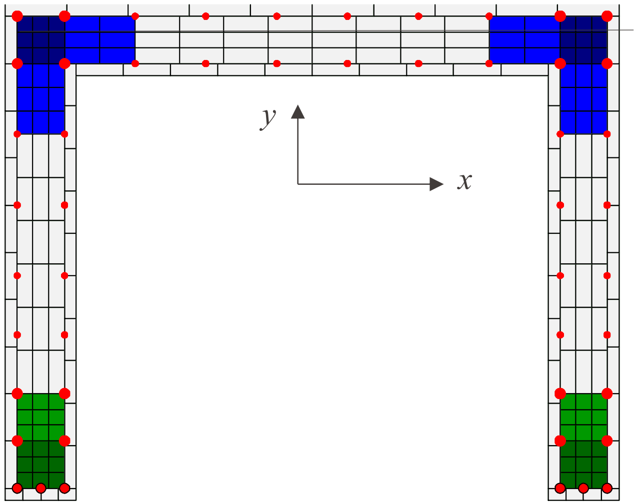
Force capacity: $$F_n = \frac{M_n}{L_v}$$

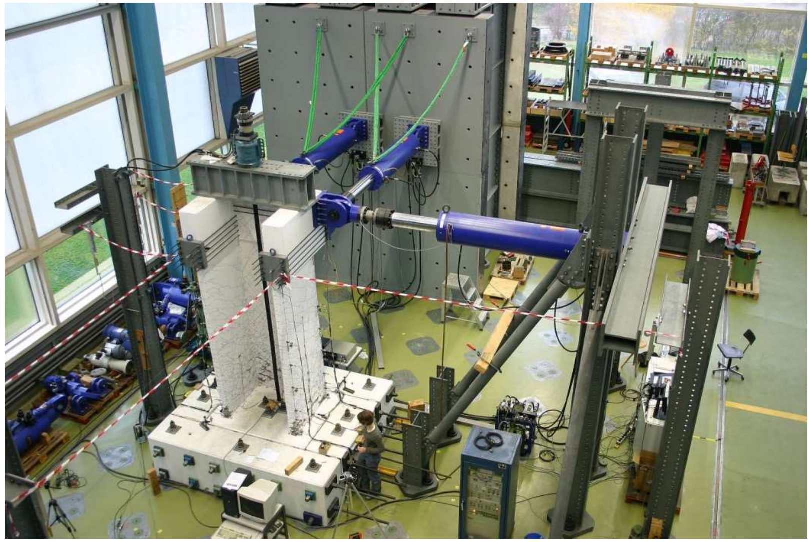
Yield displacement: $$\Delta_y = \frac{\varphi_y}{3} \cdot L_v^{\,2}$$

Ultimate displacement capacity: $$\Delta_u = \frac{\varphi_y}{3} L_v^{\,2} + \left(\varphi_u - \varphi_y\right) \cdot L_p \cdot (L_v - 0.5 \cdot L_p)$$

Plastic hinge analysis

Example: Plastic hinge analysis for a core wall





Fibre section that was used to compute the moment-curvature relationship

- White fibres: Unconfined concrete
- Green and blue fibres: Confined concrete (different degrees of confinement)
- Red fibres: Reinforcement bars

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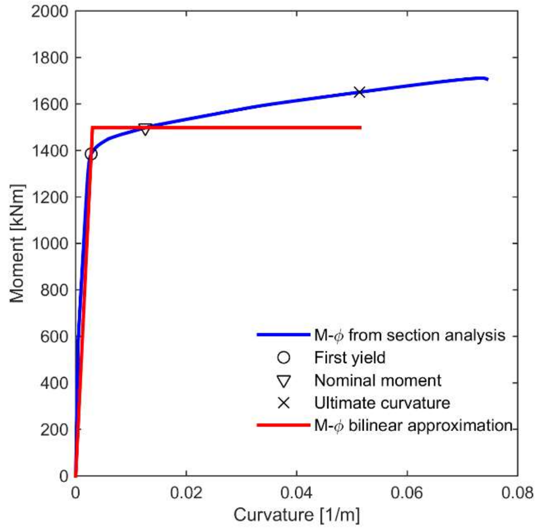
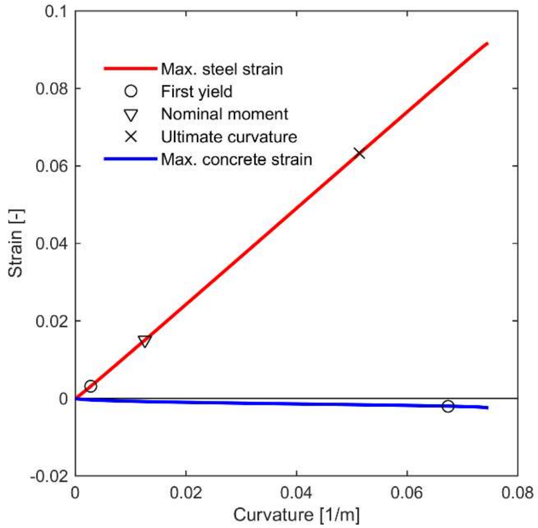
Plastic hinge analysis

Example: Moment-curvature analysis and limit curvatures





Strain capacity of reinforcement bars and confined concrete

ε_{su}=12.6 %
ε_{cu}=1.4 %

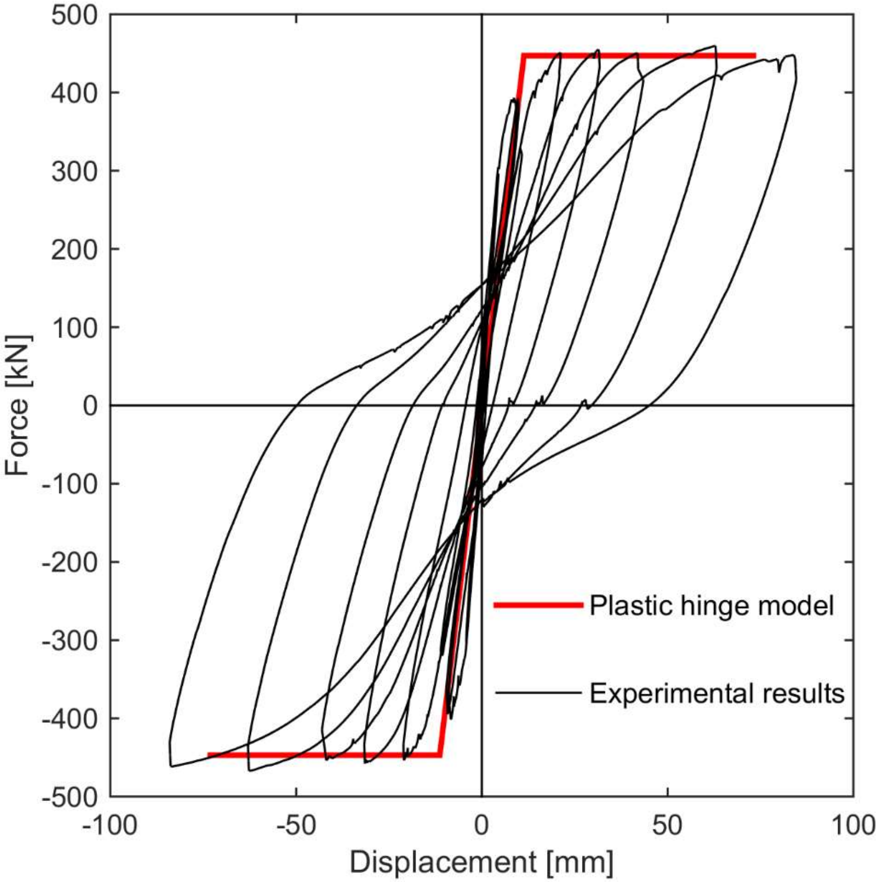
Reinf.	ε_s	$\phi(\varepsilon_s)$ [1/m]
First yield	0.24 %	0.0028
Nominal	1.50 %	0.0126
Ultimate	6.30 %	0.0514

Concrete	ε_c	$\phi(\varepsilon_c)$ [1/m]
First yield	0.2 %	0.0674
Nominal	0.4 %	
Ultimate	1.4 %	

Plastic hinge analysis

Example: Plastic hinge length according to PCK

For bending about y-axis	
L_v	3.35 m
l_w	1.3 m
d_b	12 mm
f_t	595 MPa
f_y	488 MPa
k	0.044
kL_v	0.147 m
α	0.1
αl_w	0.130 m
L_{sp}	0.129 m
$\boldsymbol{L_p}$	**0.406 m**



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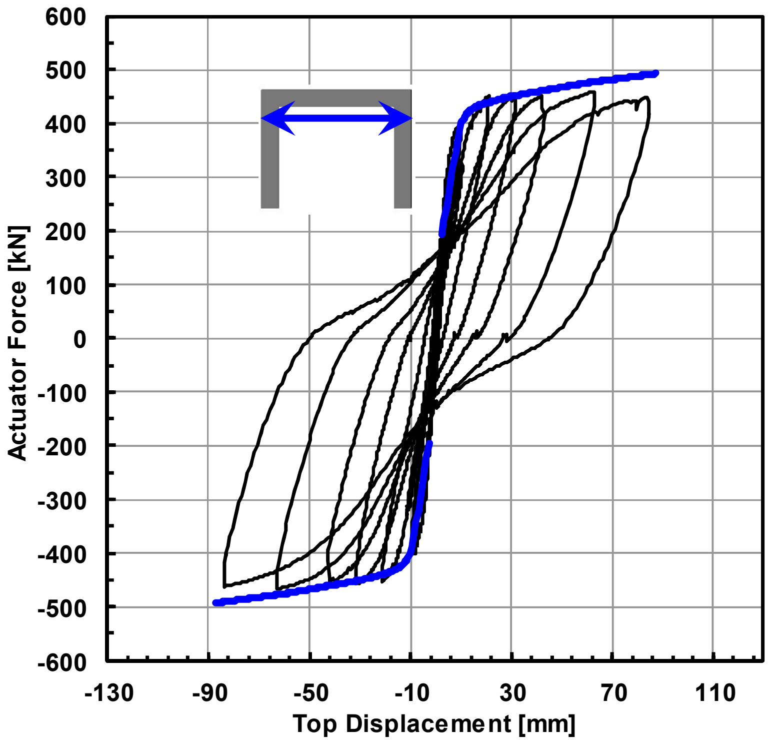
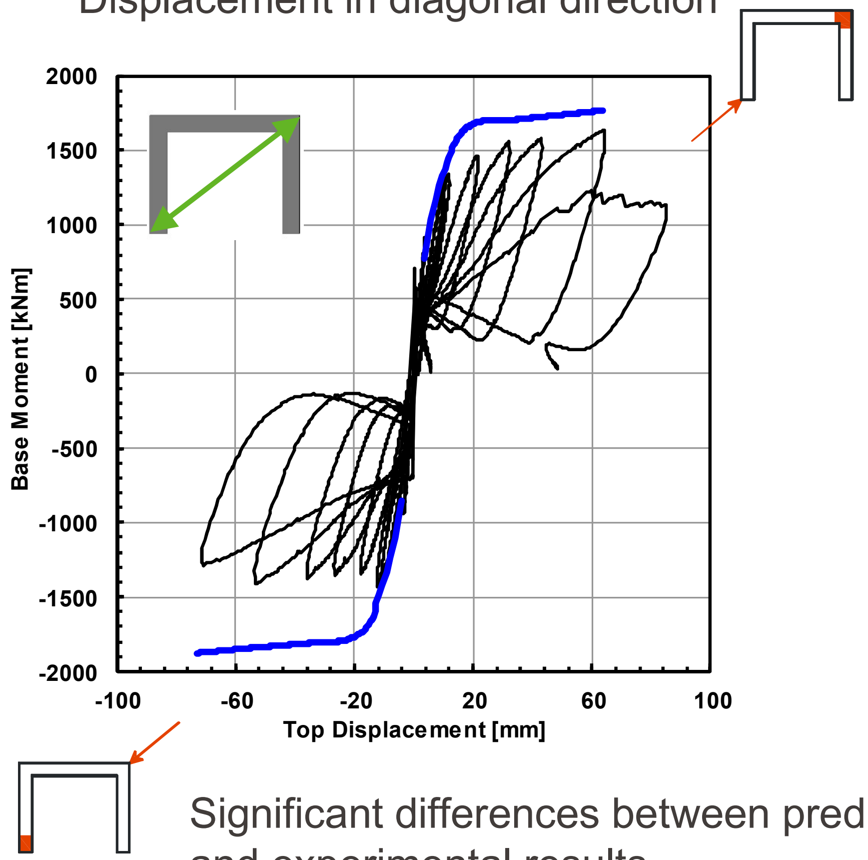
Plastic hinge analysis

Example: Remark concerning plane-section analysis for core walls

Displacement parallel to web



Displacement in diagonal direction



Plane section analysis

Experimental results

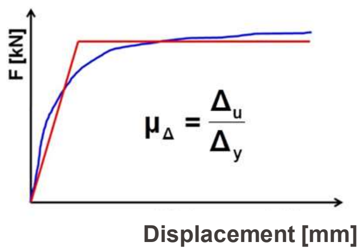
Significant differences between prediction and experimental results

< Assumption «Plane sections remain plane» does not hold any more.

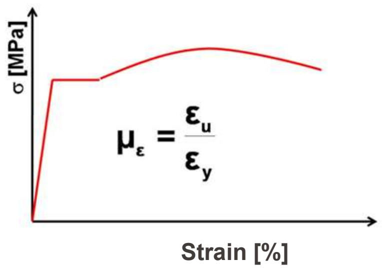
Relation between local and global ductilities

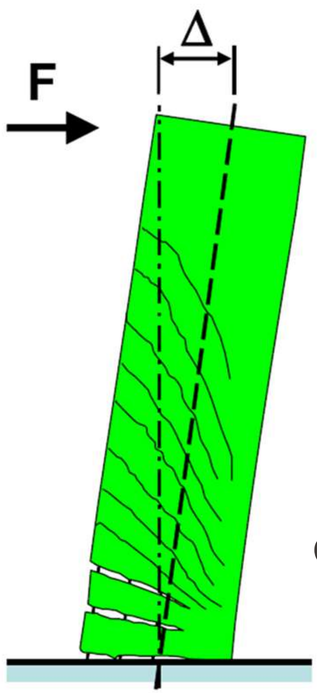
Relation between local and global ductilities

Displacement ductility

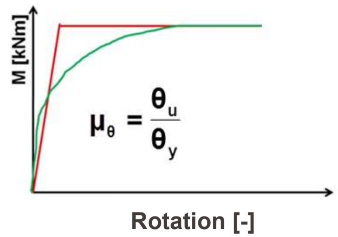


Strain ductility, e.g., strain ductility of longitudinal reinforcement

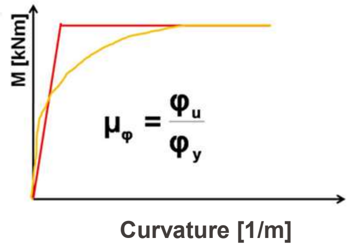




Rotation ductility



Curvature ductility



The different ductilities are not equal:

$$\mu_\Delta \neq \mu_\theta \neq \mu_\phi \neq \mu_\varepsilon$$

.. but they are related. The relation depends on the type of element (e.g. RC wall) and the geometry of the element.

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Relation between local and global ductilities

Derive an approximative relation between local and global ductilities by means of the plastic hinge method:

Yield and ultimate displacement:

$$\Delta_y = \frac{\phi_y}{3} \cdot L_v^{\ 2} \qquad \Delta_u = \frac{\varphi_y}{3} L_v^{\ 2} + \left(\varphi_u - \varphi_y\right) \cdot L_p \cdot (L_v - 0.5 \cdot L_p)$$

Local ductility: Curvature ductility

$$\mu_\phi = \frac{\phi_u}{\phi_y}$$

Global ductility: Displacement ductility

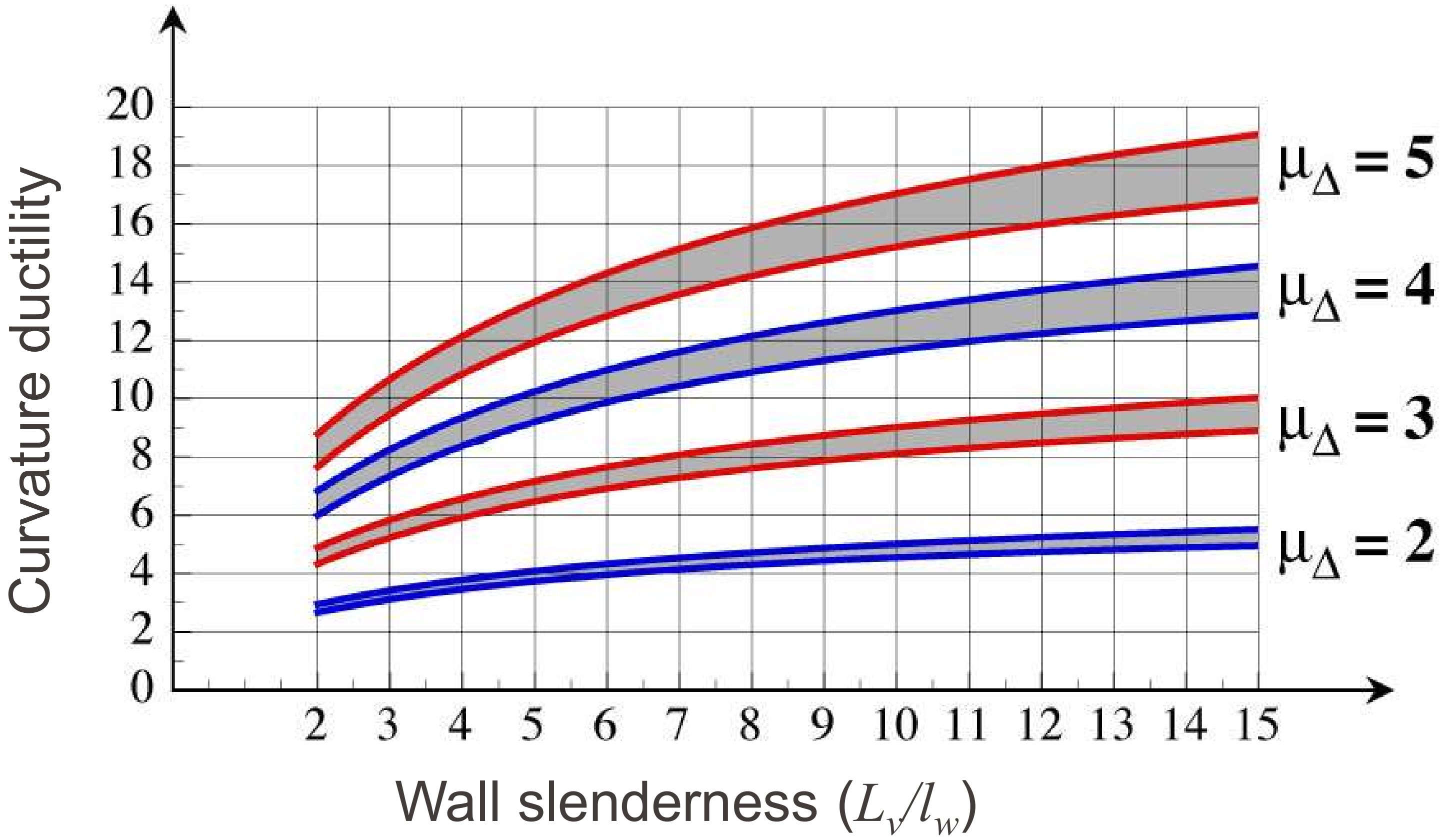
$$\mu_\Delta = \frac{\Delta_u}{\Delta_y}$$

Relation between curvature and displacement ductility:

$$\mu_\Delta \cong 1 + 3\left(\mu_\varphi - 1\right) \frac{L_p}{L_v}$$

Relation between local and global ductilities

Relationship between curvature ductility and displacement ductility



@ P. Lestuzzi

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Content

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  - Analysis of moment-curvature relationship
  - Plastic hinge method
  - Relationship between local and global ductilities

- Shortcomings of force-based design
  - How will force-based structures perform during an earthquake?
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Problems with force-based design

Problems with force-based design

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How will force-based designed buildings perform during an earthquake, i.e., how much will they be damaged?

→ We don't know!

→ Force-based design combined with capacity-design protects structures against collapse but we do not know **how much** they will be damaged.

→ Why force-based design leads to structures that perform very differently during an earthquake, is explained with the following slides.

Reference: Tom Paulay «A redefinition of the stiffness of reinforced concrete elements and its implication in seismic design»

Effective stiffness of RC members

Problem 1: Force-based design needs as input an estimate of the initial period of the building (i.e., an estimate of stiffness and mass).

For decades one assumed that …

- … the effective stiffness of a RC member is independent of its strength.
- … the yield curvature depends on the stiffness and the strength.

However, in reality, …

- … the stiffness of a RC member is dependant on its strength.
- … the yield curvature of a RC member is approximately independent of the member's strength and stiffness.

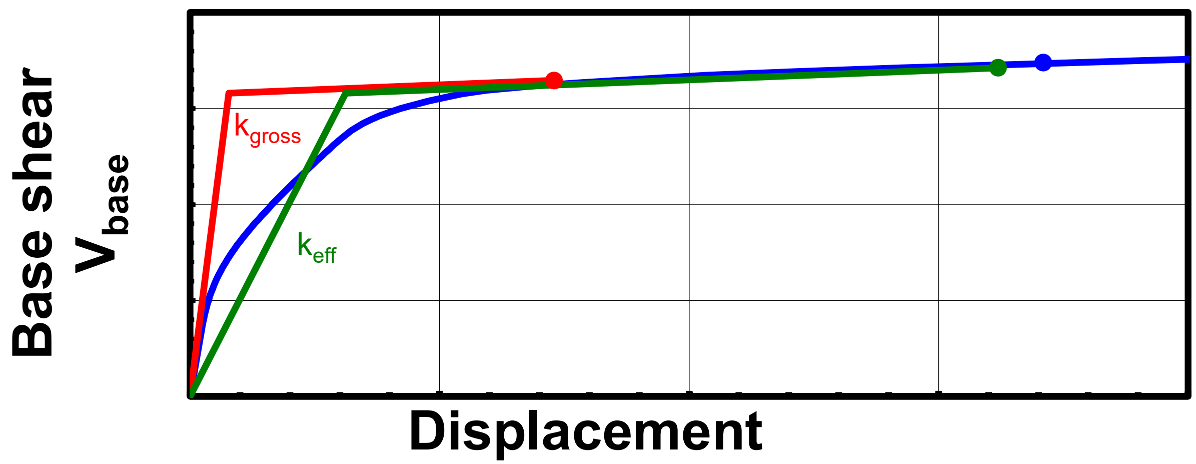
→ **This has important consequences for the force-based design.**

Effective stiffness of RC members

Stiffness of RC members

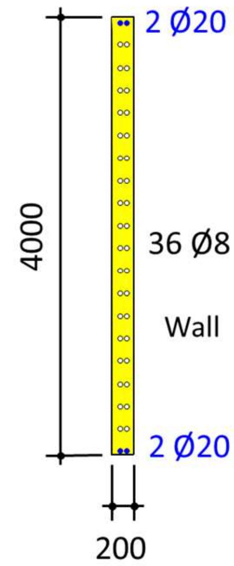
- Gross-sectional stiffness (=uncracked stiffness): This stiffness is in general not relevant for the seismic analysis. For a design earthquake (return period of 475 years), the RC structure will crack.

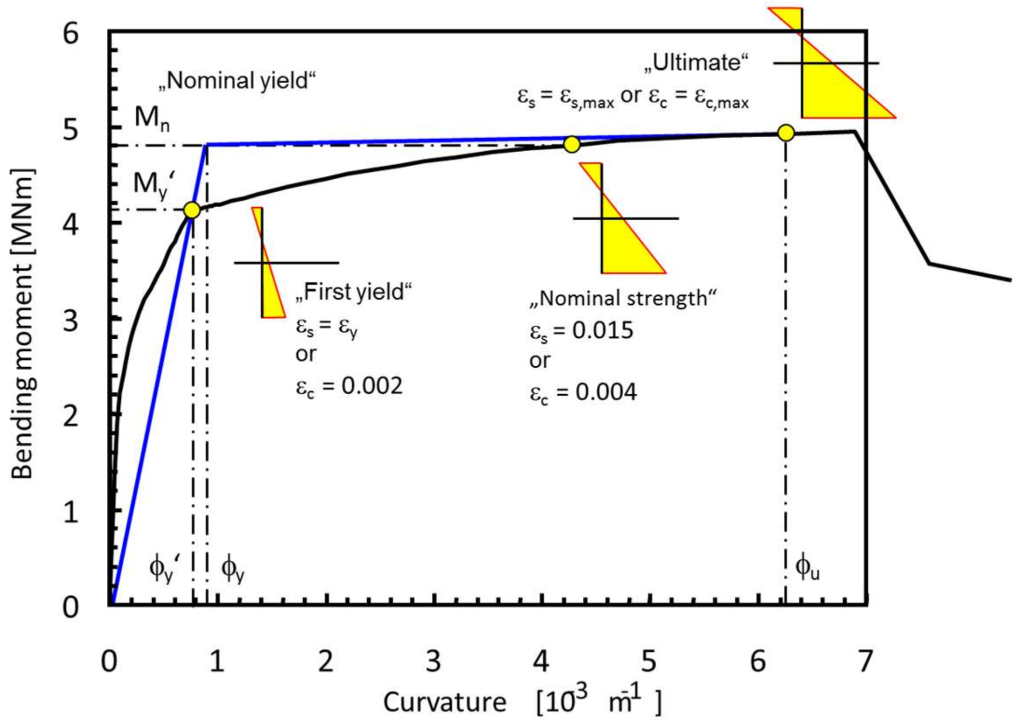
- Effective stiffness = «mean stiffness up to the point of yield» (SIA 261, 16.5.5.2). The seismic analysis is based on the effective stiffness.



Effective stiffness of RC members

Definition of the effective stiffness EI_{eff} by means of the moment-curvature relationship





$$EI_{eff} = \frac{M_n}{\phi_y}$$

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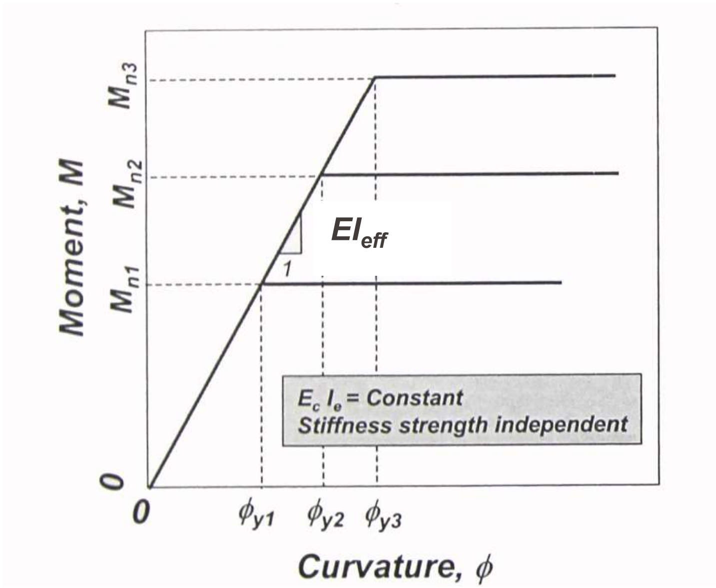
Effective stiffness of RC members

Estimation of the effective stiffness

Often: $EI_{eff} = 0.3\text{-}0.5\ EI_{gross}$

But is it correct to estimate EI_{eff} as a fixed percentage of EI_{gross}?



If yes:

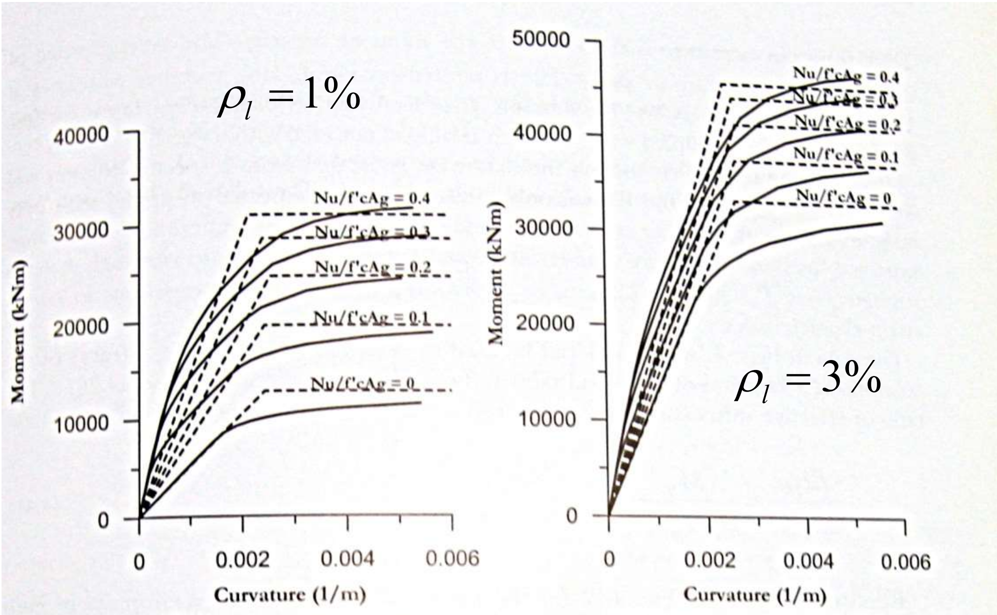
If the moment capacity of a RC wall section was varied by changing the longitudinal reinforcement content or the axial force, the effective stiffness should remain the same.

Effective stiffness of RC members

Estimation of the effective stiffness

To find the answer:

- A large number of moment-curvature analyses for different RC sections and different axial forces *N* and different longitudinal reinforcement contents ρ_l were carried out
- For each configuration, EI_{gross} and EI_{eff} were computed.
- Results for a single RC section:



One finds:

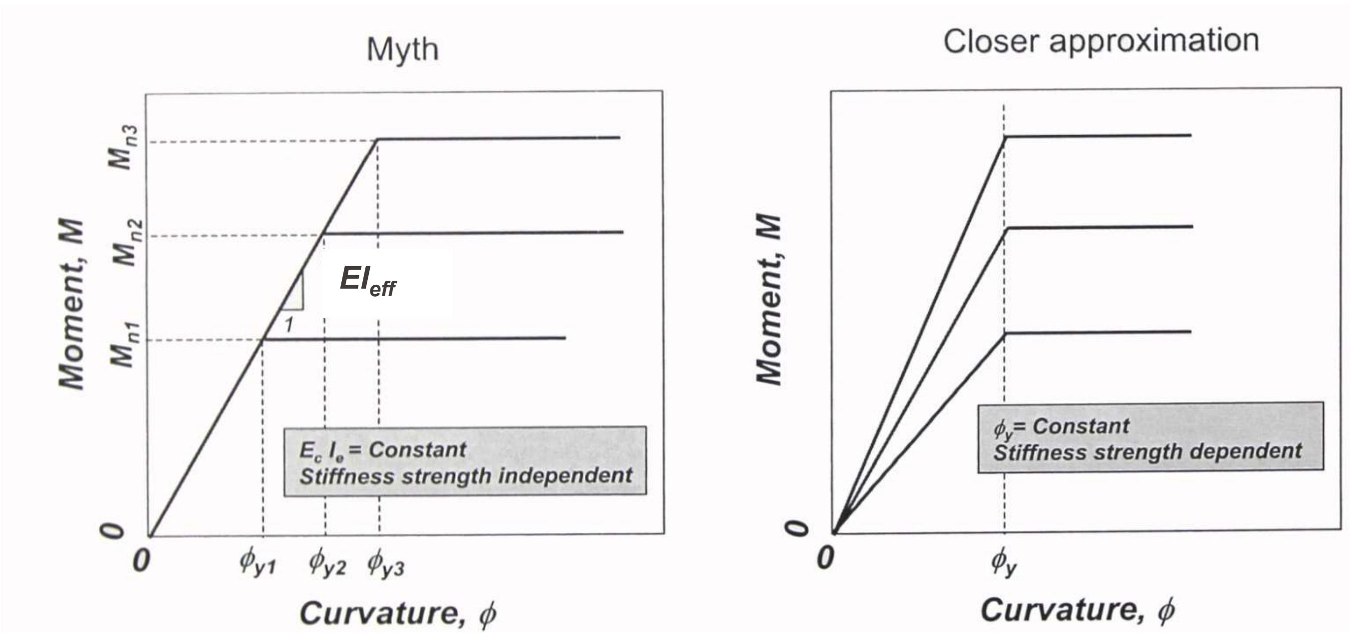
- The strength is strongly dependent on *N* and ρ_l.
- The yield curvature is almost independent of *N* and ρ_l.

Effective stiffness of RC members

Estimation of the effective stiffness

Good approximation:

- The yield curvature φ_y is independent of the section strength M_n
- The effective stiffness EI_{eff} is proportional to the section strength M_n



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Effective stiffness of RC members

Estimation of the yield curvature of RC members

- Circular column $$\phi_y = 2.25\frac{\varepsilon_y}{D}$$
- Rectangular column $$\phi_y = 2.10\frac{\varepsilon_y}{h_c}$$
- Rectangular wall $$\phi_y = 2.00\frac{\varepsilon_y}{l_w}$$
- Beam with a T-section $$\phi_y = 2.10\frac{\varepsilon_y}{h_b}$$

ε_y= Yield strain of the longitudinal reinforcement

Problems with force-based design

Recapitulation of Problem 1:

- The force-based design requires right at the beginning of the seismic design process an estimation of the fundamental period and therefore of the effective stiffness.

→ However, at this point in the design process, the strength of the members is unknown and therefore also the effective stiffness is not known.

- As a result, force-based design of RC buildings uses as very crude approximation a constant ratio of effective to gross sectional stiffness: EI_{eff} = 0.3-0.5 EI_{gross}

→ The period, one of the key input parameters of force-based design, is only very poorly estimated.

Problems with force-based design

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Further problems of force-based design:

Problem 2: The behaviour factor q is only dependent on the material (for steel structures also on the cross section class) and the structural system.

→All structures with the same structural system will be designed for the same q-factor (= same displacement ductility μ_Δ).

→The local ductility demand (curvature ductility μ_Φ) will be very different for different elements (e.g. walls of different height).

→The damage to the different elements will be very different (damage is related to local ductility demand).

Problem 3: Force-based design is based on elastic analysis but during a design-level earthquake the structure will respond inelastically.

→Distributing the design loads in a hyperstatic system based on elastic properties leads to an unfavourable distribution.

➔ The following examples illustrate these problems.

EPFL

Problems with force-based design

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Recapitulation: Behaviour factors q

RC structures in the Swiss code (SIA 262 (2013))

Class of reinforcement steel	Non-ductile design	Ductile design
Class A	q=1.5	Not permitted
Class B	q=2.0	q=3.0
Class C	q=2.0	q=4.0
Pretensioned structure	q=1.5	Not permitted

Unreinforced masonry structures: q=1.5

Steel structures: q=2-5 (dependant on the material properties, the structural system, the section class)

Problems with force-based design

2 examples that illustrate problems with force-based design:

- Bridge piers of different height — **Problem 2**
- Building with walls of different length — **Problem 3**

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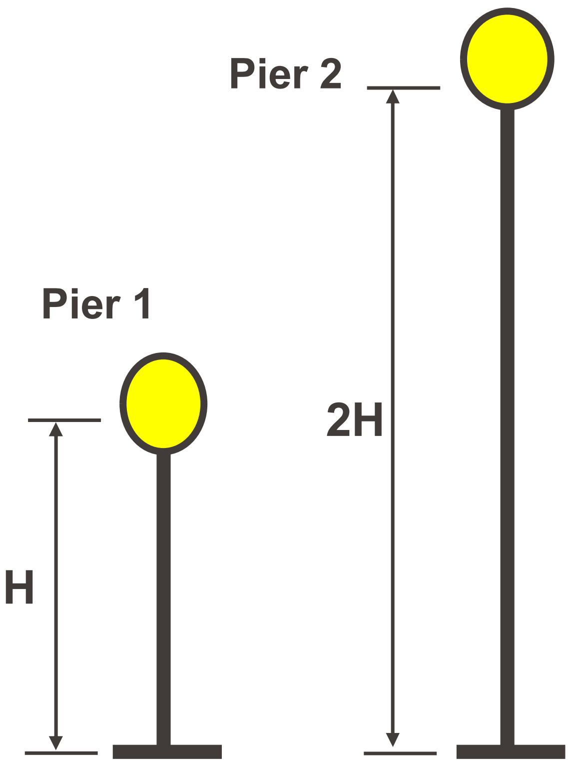
Problems with force-based design

Problem 2: q-factor just dependent on the structural system



Same cross section
Same natural period T

$q=OSR*\mu_\Delta$

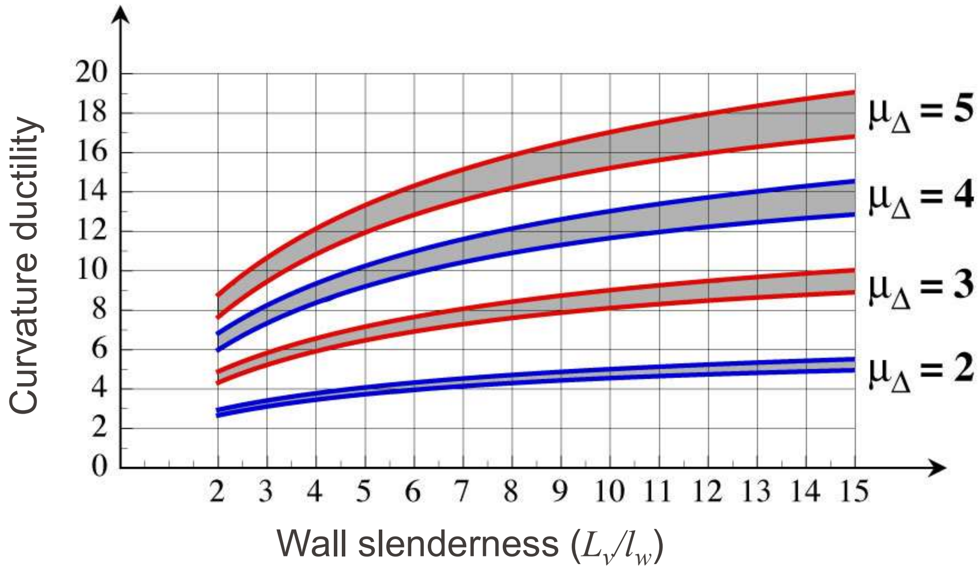
OSR = Overstrength ratio (Actual strength / design strength)

μ_Δ = Displacement ductility that the structure is expected to undergo

Example: Bridge pier with the same section but different heights are designed for the same q-factor.

Compare the curvature ductility demands that result for the two piers when both are designed for the same q-factor.

Problems with force-based design



Problems with force-based design

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Conclusions from example "Piers of different height"

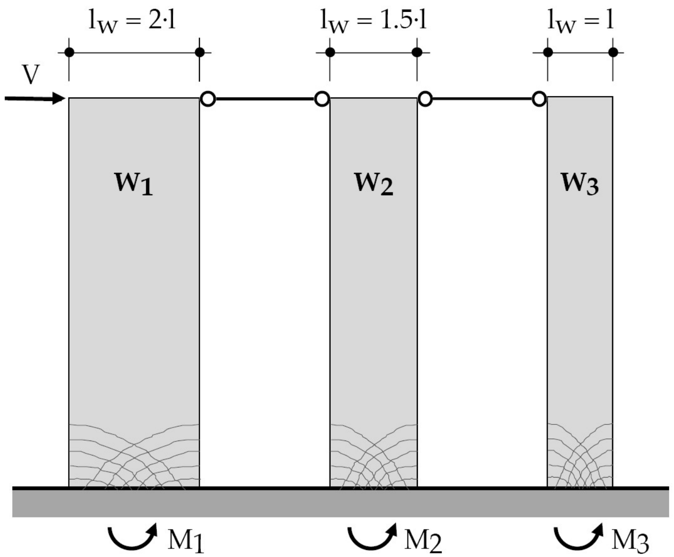
- Designing the two piers for the same q-factor means that both piers are expected to undergo the same displacement ductility demand.
- Damage relates to the curvature ductility demand μ_{Φ}
- The relationship between μ_{Φ} and μ_{Δ} depends on the geometry of the structure.

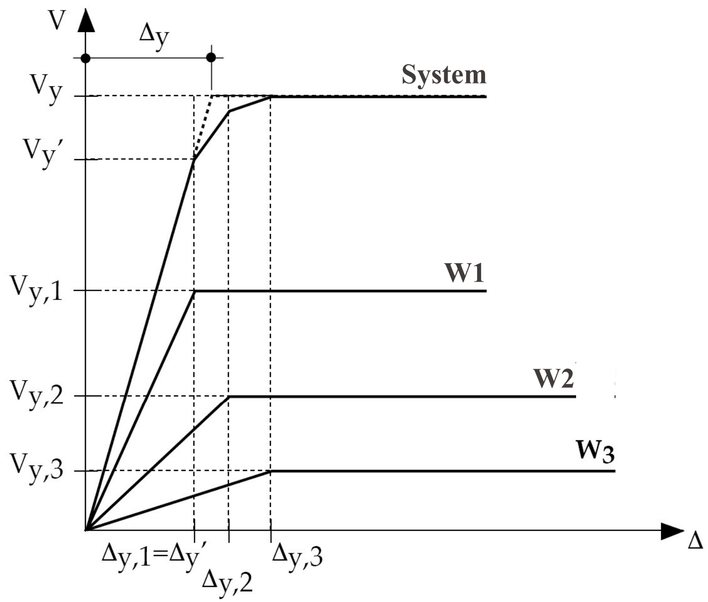
→ Since the geometry of the two structures is different, the damage to the two structures (= the performance of the two structures) will be rather different.

Problems with force-based design

Problem 3: Distribution of the horizontal force in function of k_{eff}=0.5*k_{gross} (i.e. effectively in function of k_{gross})

Example: Walls of different lengths





Compare the longitudinal and shear reinforcement ratios of the three walls that result from this elastic analysis.

Assume: All walls have the same wall width.

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Problems with force-based design

Wall	L_w	K_i/K_3	V_i/V_3	V_i/V_{tot}	M_i/M_{tot}	ρ_i/ρ_3	ρ_{Hi}/ρ_{H3}
W1	2L						
W2	1.5L						
W3	L						

Problems with force-based design

Conclusions from example “Walls of different lengths”

- The elastic analysis of hyperstatic systems does often not lead to good design solutions concerning the strength distribution between the elements.

- Paulay:
  - Forces were distributed based on elastic properties because one believed that then all elements would start yielding at the same time. However, this is not possible as the yield curvature depends only on the sections’ dimensions (and not on their strength).
  - The strength distribution should be entirely the engineer’s choice.

Problems with force-based design

Conclusions

- Force-based design suffers from the need of a period estimate right at the beginning of the design process when the strength and therefore the effective stiffness of elements is not yet known.

- Force-based design is based on elastic analysis. This does often not result in the best strength distribution in hyperstatic systems. → The distribution of the horizontal force between the different elements should be entirely the choice of the engineer.

- If force-based design is combined with capacity-design principles, the structures are nevertheless well protected against collapse for the design level earthquake.

- However, the damage to the structure (=local ductility demands) will differ significantly between structures that were all designed for the same q-factor

→ Non-uniform vulnerability of structures

→ Non-uniform performance of structures!

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